Published by Institute of Physics Publishing for SISSA

Received: *February 14, 2007*
Accepted: *February 20, 2007*
Published: *March 23, 2007*

Meta-stable supersymmetry breaking in a cooling universe

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Abstract: We look at the recently proposed idea that susy breaking can be accomplished in a meta-stable vacuum. In the context of one of the simplest models (the Seiberg-dual of super-QCD), we address the following question: if we look at this theory as it cools from high temperature, is it at all possible that we can end up in a susy-breaking meta-stable vacuum? To get an idea about the answer, we look at the free energy of the system at high temperature. We conclude that the phase-structure of the free-energy as the temperature drops, is indeed such that there is a second order phase transition in the direction of the non-susy vacuum at a finite $T = T_c^Q$. On the other hand, the potential barrier in the direction of the susy vacuum is there all the way till $T \sim 0$.

Keywords: Cosmology of Theories beyond the SM, Supersymmetry Breaking.

JHEP03(2007)107

Contents

1. Introduction

It has recently been proposed [1] that susy-breaking in a meta-stable vacuum is an interesting paradigm for model-building. The idea is that in field space, far away from the supersymmetric vacuum, we could have local minima of the effective potential with non-zero energy. These vacua can have parametrically long lifetimes because they are protected by tunneling, so using them as phenomenologically viable candidates for susy-breaking is a natural possibility. Even simple models seem to exhibit the existence of such meta-stable vacua, so it is reasonable to hope that this is a rather generic feature of supersymmetric theories. Since the original paper came out, many authors have tried to extend this idea in many directions [6 − 19].

In this paper, we will be interested in this question from a cosmological perspective. Just the fact that there exists a meta-stable vacuum in the zero-temperature field theory, is not enough to guarantee that we will end up in it (and *not* in the susy-vacuum), as the Universe cools from high-temperatures. One way to get a better understanding of this issue is to look at the phase structure of the free energy at different temperatures. This is precisely the purpose of this paper. We work in the context of one of the simplest models presented in [1], namely the S-dual of SuperQCD,[1] and look at the theory at finite temperature. The scalars in the theory are dual squarks and mesons. At zero-temperature, this theory has a supersymmetric vacuum for large values of the meson field. This is the usual susy-vacuum of SQCD, as seen from the dual picture. But the interesting thing about the dual picture is that here, we can identify new meta-stable susy-breaking vacua of the effective potential which are invisible from the pure SQCD picture. This was the key

[1] with appropriately chosen number of colors and flavors.

insight of [1]. A schematic picture of the effective potential at zero temperature is provided in figure 3.

How does this scenario change when we turn on temperature? We can get some idea about the situation by looking at the equilibrium thermal properties of this theory at finite temperature [2 − 5]. We could calculate the free energy as a function of the quark and the meson fields. At high enough temperature, we expect that free energy has a trough at the origin of field space, because the fields are massless there and the entropy is therefore higher. We could calculate the mass-matrices for the fields in the quark and meson directions to get an idea about the free-energy in those directions. Indeed, when we do this, we end up finding that as the temperature is lowered, the second order phase transition in the quark direction happens first. There is no (second order) phase transition in the meson direction, all the way down to $T \sim 0$. From these, we conclude that as the temperature drops, the Universe winds up in a susy-breaking phase.

This argument implies that the meta-stable vacuum is plausible, but we should add that this in itself is not completely conclusive. The free energy is a purely equilibrium quantity and the dynamics of the fields as they interact with a thermal bath could be more complicated. To fully understand the situation, we need to do a calculation that incorporates finite temperature effects *and* dynamics. We need to use the evolution of the field at finite temperature: we could use the imaginary-time formulation and calculate the friction-type term for the field-equation in the thermal bath. We do not report on this calculation here, but the punch-line of the preliminary calculation is that at least for some initial values of the fields, we do evolve and end up in the susy-breaking vacuum. The free energy equations give us an idea about this phase structure, as is clear from the pictures below. In each picture two separate situations are plotted together: one in the scalar mesons direction with zero squark vev and another in the squark direction with zero scalar meson vev. In 1, at high temperature the free energy drives the scalar expectation values to zero. In 2, the plot is at lower temperatures close to a threshold where the free energy in the squark direction develops a minimum away from zero, while in the meson direction it develops a potential barrier. Finally in 3, at zero temperature, the meta-stable vacuum in the squark direction remains cosmologically stable [1] due to the large potential barrier in the scalar meson direction, where the susy vacuum is located.

One important assumption is that the Seiberg transition is a post-inflation phenomenon. The reheating temperature is higher than the Seiberg transition; from the phase where the weakly coupled degrees of freedom are that of SQCD (electric phase) to the phase where the weakly coupled degrees of freedom are that of its Seiberg dual (magnetic phase). Besides, the unsolved problem of initial value of the scalar mesons after the Seiberg transition allow other possibilities that are not considered in the present work.

If initially, the scalar mesons are located far away in field space, they will execute large amplitude oscillations. Then, if both the Hubble friction and the interactions of the meson fields with the heat bath are small enough, these oscillations might bring and keep the meson fields in the vicinity of the susy vacuum.

With these assumptions the scalar fields are not in equilibrium with the rest of the universe and therefore we should not rely on an equilibrium calculation of the free energy

JHEP03(2007)107

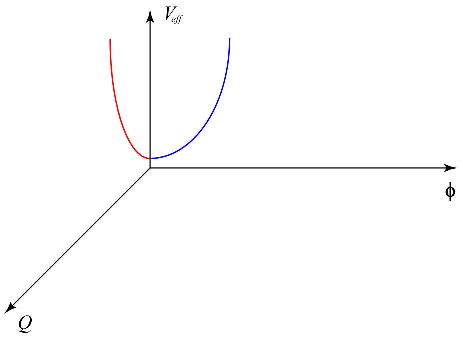

Figure 1: Effective potential for $T \gg T_c$.

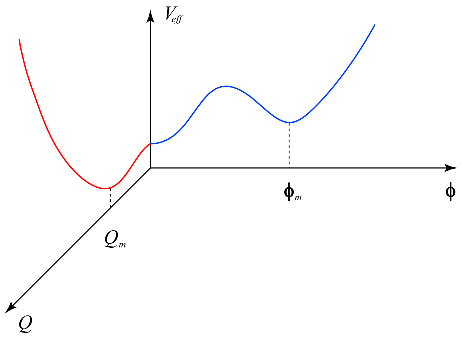

Figure 2: Effective potential for $T \sim T_c$.

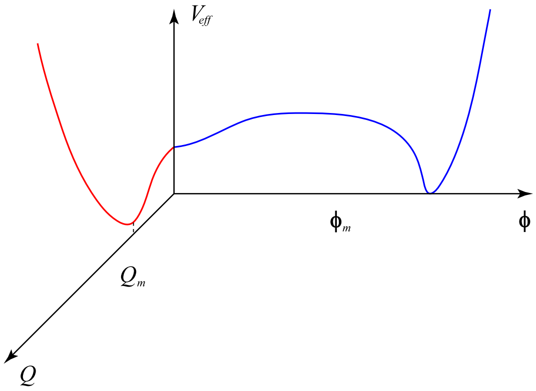

Figure 3: Effective potential for $T = 0$.

Figure 4: Evolution of the effective potential with temperature

to describe this system. It is then plausible, that as the universe cools, the meson fields find themselves trapped in the susy state. However in an adiabatic evolution of the universe it is appropriate to use thermodynamics of equilibrium to describe the system. This is what the body of this paper will make explicit.

In the next section, we introduce the model under consideration as it was described in [1]. In section 3, we begin the calculation of the equilibrium free energy of this theory along the quark and meson directions by calculating the mass matrices for the appropriate fields and expressing the free energy in the high-temperature limit. The interpretation of this object as the temperature is lowered is the subject of section 5. We draw some conclusions based on our results and end with some possibilities for future work in section 6.

2. Vacuum structure of the magnetic dual of SQCD

The meta-stable susy breaking vacuum is in a region of field space which is easier seen in the IR (magnetic) dual of SQCD. We report some of the results regarding this theory (following the original paper) in this section, but this is by no means an exhaustive review.

The superpotential for the theory (including the non-perturbative piece arising from gaugino condensation) is,

$$W = h \text{ Tr } q\Phi\tilde{q} - h\mu^2 \text{ Tr}\Phi + AN(\det\Phi)^{1/N} \tag{2.1}$$

where

$$A \equiv h^{\nu}{\Lambda_m}^{-\nu+3}, \quad \nu = N_f/N. \tag{2.2}$$

The matter content can be described by (the columns denote the SU(N) gauge and global symmetry groups):

	SU(N)	SU(N_f)	SU(N_f)	U$(1)_B$	U$(1)'$	U$(1)_R$
Φ	1	$\square$	$\overline{\square}$	0	−2	2
q	$\square$	$\overline{\square}$	1	1	1	0
$\tilde{q}$	$\overline{\square}$	1	$\square$	−1	1	0

The second piece in the superpotential breaks the SU(N_f)×SU(N_f) down to a diagonal SU(N_f) subgroup. In our notation, the q stand for the (dual) quarks, the Φ are gauge singlet mesons and Λ_m is the dynamically generated scale (the scale of the Landau pole) for the IR theory. The number of colors of the magnetic theory N, and the number of flavors N_f, satisfy $N_f > 3N$ so that the theory is IR free. Notice that all our notation is defined in terms of the dual theory and *not* directly in terms of the microscopic theory (which is SQCD).

For small meson fields, the non-perturbative term can be dropped, and we can calculate the moduli space of susy-breaking tree-level vacua. These are at

$$\Phi = \begin{pmatrix} 0 & 0 \\ 0 & \varphi \end{pmatrix}, \quad q = \begin{pmatrix} Q \\ 0 \end{pmatrix}, \quad \tilde{q} = \begin{pmatrix} \tilde{Q} & 0 \end{pmatrix}, \tag{2.3}$$

with $\tilde{Q}Q = \mu^2 \mathbb{1}_N$ where $\tilde{Q}$ and Q are $N \times N$ matrices and φ is a $(N_f - N) \times (N_f - N)$ matrix. In this vacum the scalar potential has the value

$$V_{\min} = (N_f - N)|h^2\mu^4|$$

The point of maximum global symmetry in this moduli space, up to gauge transformation and flavor rotation, is

$$\varphi = 0, \quad Q = \tilde{Q} = \mu\mathbb{1}_N, \tag{2.4}$$

The interesting point is that along with these meta-stable vacua, far away in the moduli-space at large meson values, we have supersymmetric vacua that arise from the extremization of the superpotential. These are of course the familiar SUSY vacua of SQCD, seen from the dual picture.

Putting all these together, the vacuum structure of the IR dual of SQCD looks schematically like figure 3. The susy-breaking vacuum is in a local trough (in all directions, even though we indicate only the quark direction), so it is meta-stable and it is cosmologically stable due to a large potential barrier.

JHEP03(2007)107

3. Free energy

Our aim now, is to look at the above theory, and calculate the finite temperature effective potential (free energy) up to one loop. We want to see how the picture above changes when we turn on temperature. This will give us a clue about the possible phase transitions that could happen, as the universe cooled. The standard procedure for calculating the finite-temperature effective potential is to shift the relevant background fields (in our case the scalar quarks and mesons) and use the resulting quadratic pieces in the action to do the computation. So essentially, we need to know what the masses[2] of the various fields are, as a result of the shifts in the backgrounds. It is the calculation of these mass matrices, that we undertake next. For the sake of simplicity, we will take the parameters h and μ as well as the shifts in the background fields to be real numbers.

3.1 Mass matrices in the meson directions

We first calculate the masses of the various fields when a background field is turned on in the meson direction, with the scalar quarks set to zero. In the next subsection we will turn on the squarks and turn off the mesons. Once we know the mass matrices in both these directions, we can draw some conclusions about the free energy and the phase structure as a function of temperature.

Decomposing the meson chiral superfield into a background, trace and a traceless part,

$$\Phi = \Big(\varphi + \frac{\Phi_0}{\sqrt{N_f}}\Big)\mathbb{1} + \widehat{\Phi}, \tag{3.1}$$

we can expand the nonperturbative term in the superpotential:

$$\begin{aligned} AN(\det\Phi)^{1/N} &= AN\varphi^{\nu}\exp\left(\tfrac{1}{N}\mathrm{Tr}\ln(\mathbb{1}+\tfrac{\Phi_0}{\sqrt{N_f}\varphi}\mathbb{1}+\tfrac{\widehat{\Phi}}{\varphi})\right) \\ &\sim AN\varphi^{\nu}\exp\left(\tfrac{1}{N}(\tfrac{\Phi_0\sqrt{N_f}}{\varphi}-\tfrac{\Phi_0^2}{2\varphi^2}-\tfrac{\mathrm{Tr}(\widehat{\Phi}^2)}{2\varphi^2}+\tfrac{\Phi_0^3}{3\sqrt{N_f}\varphi^3})\right) \\ &\sim A\varphi^{\nu}\Bigg(N+\frac{\Phi_0\sqrt{N_f}}{\varphi}+\frac{(\nu-1)\Phi_0^2}{2\varphi^2}-\frac{\mathrm{Tr}(\widehat{\Phi}^2)}{2\varphi^2}+ \\ &\qquad\qquad +\frac{\Phi_0^3(\nu-1)(\nu-2)}{6\sqrt{N_f}\varphi^3}-\frac{(\nu-2)\Phi_0\mathrm{Tr}(\widehat{\Phi}^2)}{2\sqrt{N_f}\varphi^3}\Bigg) \end{aligned}$$

From this, we can read off the terms that contribute to the fermion masses in the Lagrangian:

$$\mathcal{L} \supset h\varphi\ \mathrm{Tr}(\psi_q\psi_{\tilde{q}}) + \frac{A\varphi^{\nu-2}(\nu-1)}{2}\psi_{\phi_0}\psi_{\phi_0} - \frac{A\varphi^{\nu-2}}{2}\ \mathrm{Tr}(\psi_{\hat{\phi}}\psi_{\hat{\phi}}) + h.c. \tag{3.2}$$

showing that there are $N\times N_f$ Dirac fermions with mass $h\varphi$; 1 Majorana fermion with mass $A\varphi^{\nu-2}(\nu-1)$ and (N_f^2-1) Majorana fermions with mass $A\varphi^{\nu-2}$.

[2] To be more precise, we should say that we are after the coefficients of the pieces in the action that are quadratic in fields, we will call them masses even though we are not necessarily looking at the extremum of the potential.

# of weyl fermions	fermion fields	mass
$2N \times N_f$	$\psi_q,\ \psi_{\tilde{q}}$	$h\varphi$
1	ψ_{ϕ_0}	$A\varphi^{\nu-2}(\nu-1)$
(N_f^2-1)	$\psi_{\hat{\phi}}$	$A\varphi^{\nu-2}$

Table 1: fermions masses, meson direction.

# of real scalars	scalar fields	$mass^2$
$2(N_f^2-1)$	$\widehat{\phi}$	$\lvert A\rvert^2\varphi^{2\nu-4}$
1	$Im\ \phi_0$	$\lvert A\rvert^2\varphi^{2\nu-4}(\nu-1)^2-$ $-Re\left[(A\varphi^{\nu-1}-h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]$
1	$Re\ \phi_0$	$\lvert A\rvert^2\varphi^{2\nu-4}(\nu-1)^2+$ $+Re\left[(A\varphi^{\nu-1}-h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]$
$N \times N_f$	$Re(q+\tilde{q}^t)/\sqrt{2}$	$h^2\varphi^2+Re(A\varphi^{\nu-1}-h\mu^2)h$
$N \times N_f$	$Im(q+\tilde{q}^t)/\sqrt{2}$	$h^2\varphi^2$
$N \times N_f$	$Re(q-\tilde{q}^t)/\sqrt{2}$	$h^2\varphi^2-Re(A\varphi^{\nu-1}-h\mu^2)h$
$N \times N_f$	$Im(q+\tilde{q}^t)/\sqrt{2}$	$h^2\varphi^2$

Table 2: real scalar squared masses, meson direction.

To calculate the scalar masses from the scalar potential, we first compute the F-terms.

$$F_{\phi_0} = \frac{h}{\sqrt{N_f}}\,\mathrm{Tr}(q\tilde{q}) - h\mu^2\sqrt{N_f} + A\varphi^{\nu}\left(\frac{\sqrt{N_f}}{\varphi}+\frac{\nu-1}{\varphi^2}\phi_0+\frac{(\nu-1)(\nu-2)}{2\varphi^3\sqrt{N_f}}\phi_0^2\right) \quad (3.3)$$

$$F_{\hat{\phi}} = h\tilde{q}q + A\varphi^{\nu}\left(-\frac{\widehat{\phi}^t}{\varphi^2}-\frac{(\nu-2)}{\sqrt{N_f}\varphi^3}\phi_0\widehat{\phi}^t\right) \quad (3.4)$$

$$F_q = h(\phi\tilde{q})^t \supset h(\varphi\tilde{q})^t \quad (3.5)$$

$$F_{\tilde{q}} = h(\phi q)^t \supset h(\varphi q)^t \quad (3.6)$$

Consequently, the scalar potential will have the following quadratic terms:

$$V_{\text{scalar}} \supset (A\varphi^{\nu-1}-h\mu^2)\left[\mathrm{Tr}(q\tilde{q})h+\frac{(\nu-1)(\nu-2)A\varphi^{\nu-3}}{2}\phi_0^2\right]+c.c.+ \quad (3.7)$$

$$|A|^2\varphi^{2\nu-4}((\nu-1)^2|\phi_0|^2+|\widehat{\phi}|^2)+h^2\varphi^2(|\tilde{q}|^2+|q|^2) \quad (3.8)$$

where the scalar masses are extracted and shown in table 2.

3.2 Mass matrices in the quark directions

The mass matrices in the quark directions are more complicated than in the meson directions because there are contributions from the D-terms. We start by classifying the various sectors according to their transformation properties under the global symmetries. We take

# of real scalars	scalar fields	$mass^2$
$4N_m \times N_e$	ϕ_{12} and ϕ_{21}	h^2Q^2
$N_m \times N_e$	$Re(q_2 + \tilde{q}_2^t)/\sqrt{2}$	$h^2(Q^2 + \mu^2)$
$N_m \times N_e$	$Re(q_2 - \tilde{q}_2^t)/\sqrt{2}$	$h^2(Q^2 - \mu^2)$
$N_m \times N_e$	$Im(q_2 + \tilde{q}_2^t)/\sqrt{2}$	h^2Q^2
$N_m \times N_e$	$Im(q_2 - \tilde{q}_2^t)/\sqrt{2}$	h^2Q^2

Table 3: real scalar squared masses, sectors 2 and 3.

the shifts in the form: $\langle\tilde{q}_1\rangle = \langle q_1\rangle = Q\mathbb{1}$, Q real. The column vectors are $N \times N_e$ where $N_e = N_f - N$. The subscript stands for electric. N_e is the same as N_c, the number of colors in the original (microscopic) theory, namely SQCD, but we prefer to think of it purely in terms of the dual theory. On a similar note, N is actually N_m, with the subscript standing for magnetic.

There are four sectors that do not mix with each other in the Lagrangian. We will use this fact to our advantage in calculating the mass matrices. These sectors are:

1. $N_e \times N_e$: ϕ_{22},
2. $N_e \times N_m$: $\tilde{q}_2, \phi_{21}$,
3. $N_m \times N_e$: q_2, ϕ_{12},
4. $N_m \times N_m$: $q_1, \tilde{q}_1, \phi_{11}, V$.

The V in the last line is the vector superfield, it gets massive through a Higgs mechanism.

Now we look at the masses of the various fields sector by sector. To start off, in the $N_e \times N_e$ sector, there are $N_e^2 \times$ (complex scalars + Weyl Fermions), and all of them remain entirely massless (at tree level). But it is in this sector that supersymmetry is broken by a positive contribution in the scalar potential. Decomposing Φ_{22} into trace and traceless parts, we get

$$\Phi_{22} = \frac{\Phi_{22}^0}{\sqrt{N_e}}\mathbb{1} + \widehat{\Phi}_{22}; \quad F_{\phi_{22}^0} \supset -h\mu^2\sqrt{N_e}.$$

Hence

$$V_{\text{scalar}} \geq N_e h^2 \mu^4.$$

It is easiest to deal with the two mixed electric/magnetic sectors (2 and 3) together. Together, there are $2N_eN_m \times$ (2 complex scalars + 1 Dirac fermion). The fermionic masses arise from terms like

$$hQ\ \mathrm{Tr}(\Psi_{12}^{\phi}\Psi_2^{q}) + hQ\ \mathrm{Tr}(\Psi_2^{\tilde{q}}\Psi_{21}^{\phi}) + h.c., \tag{3.9}$$

and these give rise to a Dirac mass of hQ. To calculate the bosonic masses, we need the scalar potential in the quark direction which can be calculated easily enough from the superpotential and the F-terms. The F-terms $F_{q_1}, F_{\tilde{q}_1}$ do not give rise to scalar masses in sectors 2 and 3 because the scalars from these sectors have zero vevs. The relevant

JHEP03(2007)107

# fields	fields	mass
$N_m^2 - 1$	$A^{\mu,a}$	$2gQ$
$N_m^2 - 1$	λ^a	$2gQ$
$N_m^2 - 1$	traceless $(\Psi_1^q - \Psi_1^{\tilde{q}})/\sqrt{2}$	$2gQ$
$2N_m^2$	Ψ_{11}^{ϕ}	$hQ\sqrt{2}$
$2N_m^2$	$(\Psi_1^q + \Psi_1^{\tilde{q}})/\sqrt{2}$	$hQ\sqrt{2}$

Table 4: vector boson and fermion masses, sector 4.

non-vanishing ones are

$$
\begin{aligned}
F_{\phi_{12}} &= h(\tilde{q}_2 q_1)^t \supset hQ(\tilde{q}_2)^t && (3.10)\\
F_{\phi_{21}} &= h(\tilde{q}_1 q_2)^t \supset hQ(q_2)^t && (3.11)\\
F_{\phi_{22}} &= h(\tilde{q}_2 q_2)^t - h\mu^2 \times \mathbb{I} && (3.12)\\
F_{\tilde{q}_2} &= h(q_1\phi_{12} + q_2\phi_{22}) \supset hQ\phi_{12} && (3.13)\\
F_{q_2} &= h(\phi_{21}\tilde{q}_1 + \phi_{22}\tilde{q}_2) \supset hQ\phi_{21} && (3.14)
\end{aligned}
$$

Therefore, the scalar potential contains the quadratic terms:

$$V_{\text{scalar}} \supset h^2 \left(Q^2|\tilde{q}_2|^2 + Q^2|q_2|^2 - \mu^2 \ \mathrm{Tr}(\tilde{q}_2 q_2 + h.c.) + Q^2|\phi_{12}|^2 + Q^2|\phi_{21}|^2\right) \tag{3.15}$$

where in our notation modulus squared of matrices means trace over the product of the matrix and its adjoint. As we see, $2N_m \times N_e$ complex scalars (ϕ_{12} and ϕ_{21}) get squared mass h^2Q^2, $2N_m \times N_e$ real scalars $(Re(q_2 \pm \tilde{q}_2^t)/\sqrt{2})$ split their masses into $h^2(Q^2 \pm \mu^2)$ and another $2N_m \times N_e$ real scalars $(Im(q_2 \pm \tilde{q}_2^t)/\sqrt{2})$ get mass h^2Q^2. See table (3).

Now we turn to sector 4. First, we separate out the background:

$$q_1 = Q\mathbb{I} + \widehat{q}_1 \ ; \ \ \tilde{q}_1 = Q\mathbb{I} + \widehat{\tilde{q}}_1 \tag{3.16}$$

Some of the fermion masses arise from the terms

$$g\sqrt{2}Q \ \mathrm{Tr}(\lambda\Psi_1^q) - g\sqrt{2}Q \ \mathrm{Tr}(\lambda\Psi_1^{\tilde{q}}) + h.c. \tag{3.17}$$

where gauginos λ^a, and the traceless $(\Psi_1^q - \Psi_1^{\tilde{q}})/\sqrt{2}$ have equal masses $2gQ$.

From the Kähler potential, the vector bosons A_μ get a mass $2gQ$. The traceless part of $Im\left((\widehat{q}_1 - \widehat{\tilde{q}}_1)/\sqrt{2}\right)$ is gauged away and the traceless part of the scalars $Re\left((\widehat{q}_1 - \widehat{\tilde{q}}_1)/\sqrt{2}\right)$ get their masses from the F-terms (to be calculated) and from the D-terms. The contribution to this squared mass from the D-terms can be read off from

$$
\begin{aligned}
\frac{g^2}{2}\sum_a(\mathrm{Tr}(q_1^\dagger t^a q_1 - \tilde{q}_1 t^a \tilde{q}_1^\dagger))^2 &\supset \frac{g^2}{2}\sum_a(Q\mathrm{Tr}(t^a(\widehat{q}_1 - \widehat{\tilde{q}}_1 + \widehat{q}_1^\dagger - \widehat{\tilde{q}}_1^\dagger))^2\\
&= \frac{g^2}{2}\sum_a(Q\mathrm{Tr}(2Re[t^a(\widehat{q}_1 - \widehat{\tilde{q}}_1)]))^2\\
&= 4g^2Q^2\sum_a\left(\mathrm{Tr}(Re[t^a(\widehat{q}_1 - \widehat{\tilde{q}}_1)/\sqrt{2}])\right)^2 && (3.18)
\end{aligned}
$$

# of real bosons	fields	$mass^2$
N_m^2	$Re\ (\widehat{q_1}+\widehat{\tilde{q}}_1)/\sqrt{2}$	$h^2(3Q^2-\mu^2)$
$2N_m^2$	ϕ_{11}	$2h^2Q^2$
N_m^2	$Im(\widehat{q_1}+\widehat{\tilde{q}}_1)/\sqrt{2}$	$2h^2Q^2$
N_m^2-1	traceless $Re(\widehat{q_1}-\widehat{\tilde{q}}_1)/\sqrt{2}$	$h^2(\mu^2-Q^2)+4g^2Q^2$
1	Tr $\left(Re(\widehat{q_1}-\widehat{\tilde{q}}_1)/\sqrt{2}\right)$	$h^2(\mu^2-Q^2)$
1	Tr $\left(Im(\widehat{q_1}-\widehat{\tilde{q}}_1)/\sqrt{2}\right)$	0

Table 5: real squared masses for bosons, sector 4.

to be $4g^2Q^2$. The scalar $Im\left(\mathrm{Tr}(q_1-\tilde{q}_1)\right)/\sqrt{2}$ and the fermion $\mathrm{Tr}(\Psi_1^q-\Psi_1^{\tilde{q}})/\sqrt{2}$ remain massless at tree level. But $Re(\mathrm{Tr}(q_1-\tilde{q}_1))/\sqrt{2}$ receives a contribution from the F-terms.

The shifts in q_1 and $\tilde{q}_1$ give rise to terms of the form

$$hQ\Psi_{11}^{\phi}(\Psi_1^q+\Psi_1^{\tilde{q}})+h.c.$$

From these, the $2N_m^2$ Weyl fermions (Ψ_{11}^{ϕ}) and $(\Psi_1^q+\Psi_1^{\tilde{q}})/\sqrt{2}$ acquire a mass of $hQ\sqrt{2}$ each.

Their respective scalar superpartners acquire mass through the scalar potential. Writing the relevant terms in the F-terms, using (3.16), we have

$$F_{\phi_{11}}=h\left((Q^2-\mu^2)\mathbb{I}+Q\sqrt{2}\frac{\widehat{q_1}+\widehat{\tilde{q}}_1}{\sqrt{2}}+\widehat{\tilde{q}}_1\widehat{q_1}\right)$$

and $F_{q_1}\supset hQ\phi_{11}$, $F_{\tilde{q}_1}\supset hQ\phi_{11}$. Hence, the quadratic terms in the scalar potential are,

$$V_{\mathrm{scalar}}\supset h^2\left(2Q^2\left|\phi_{11}\right|^2+2Q^2\left|\frac{\widehat{q_1}+\widehat{\tilde{q}}_1}{\sqrt{2}}\right|^2+(Q^2-\mu^2)\left(\ \mathrm{Tr}(\widehat{\tilde{q}}_1\widehat{q_1})+c.c\right)\right)\tag{3.19}$$

This shows that among the real scalars, $2N_m^2$ get squared masses $2h^2Q^2$, N_m^2 get squared masses $h^2(3Q^2-\mu^2)$, and N_m^2 get squared masses $2h^2Q^2$. The corresponding fields are ϕ_{11}, $Re(\widehat{\tilde{q}}_1+\widehat{q_1})/\sqrt{2})$ and $Im(q_1+\tilde{q}_1)/\sqrt{2}$ respectively. The terms $Re(\widehat{\tilde{q}}_1-\widehat{q_1})/\sqrt{2}$ get mass from above and from the D-terms, splitting the field matrix into a trace part with mass $h^2(\mu^2-Q^2)$ and N_m^2-1 traceless components with mass $h^2(\mu^2-Q^2)+4g^2Q^2$.

The vector boson, gaugino and fermion masses are presented in table (4), and the scalar masses are in table (5).

3.3 Effective potential

The effective potential at finite temperature is the free energy of a system in a thermal bath. In thermal equilibrium, there is no time dependence and the different phases correspond to local minima of the free energy.

JHEP03(2007)107

The finite temperature effective potential up to one loop is given by [4]

$$V(\phi_{cl}) = V_{\rm tree}(\phi_{cl}) + V_1^0(\phi_{cl}) + V_1^T(\phi_{cl})$$

where $V_{\rm tree}(\phi_{cl})$ is the classical piece. The one-loop correction, $V_1^T(\phi_{cl})$, for a generic theory is:

$$\begin{aligned} V_1^T(\phi_{cl}) \approx & -\frac{\pi^2 T^4}{90}\left(N_B + \frac{7}{8}N_F\right) + \frac{T^2}{24}\left[\sum_i (M_S^2)_i + 3\sum_i (M_V^2)_i + \sum_i (M_F^2)_i\right] \\ & -\frac{T}{12\pi}\left[\sum_i (M_S^3)_i + 3\sum_i (M_V^3)_i\right] + \dots \end{aligned} \quad (3.20)$$

and $V_1^0(\phi_{cl})$ is the zero-temperature piece, $(M_S)_i,\ (M_V)_i$ and $(M_F)_i$ rae mass-matrix eigenvalues for the real scalars, vectors and Weyl fermions respectively, and $N_B = N_F$ is the number of bosonic/fermionic degrees of freedom, paired by supersymmetry. What we have done here is to follow the standard practice and split off the one-loop, finite-temperature effective potential into a part that is independent of temperature (and therefore is the same as the zero-temperature effective potential) and then do a high-temperature ($T \gg$ masses) expansion on the remaining (temperature-dependant) piece. The zero temperature piece is calculated using the usual supersymmetric generalization of the Coleman-Weinberg formula [5]:

$$V_1^0 = \frac{1}{64\pi^2}{\rm STr}\,\mathcal{M}^4 \log\frac{\mathcal{M}^2}{\Lambda^2}, \quad (3.21)$$

where $\mathcal{M}$ stands for the full mass-matrix, with the supertrace providing the negative sign for the fermionic terms.

Armed with the above expressions and the mass-matrices from the last section, its easy to calculate the finite-temperature effective potential. In the following, we only keep the terms that are quartic and quadratic in temperature. In the case of background fields in the meson direction, we find:

$$V_{\rm tree}(\varphi) = N_f|(A\varphi^{\nu-1} - h\mu^2)|^2, \quad (3.22)$$

$$\begin{aligned} V_1^0(\varphi) = & \frac{1}{64\pi^2}\Big(-2|A|^4\varphi^{4\nu-8}(\nu-1)^4 \log\frac{|A|^2\varphi^{2\nu-4}(\nu-1)^2}{\Lambda^2} + \\ & +[|A|^2\varphi^{2\nu-4}(\nu-1)^2 - Re\left[(A\varphi^{\nu-1} - h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]]^2 \times \\ & \times \log\frac{|A|^2\varphi^{2\nu-4}(\nu-1)^2 - Re\left[(A\varphi^{\nu-1} - h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]}{\Lambda^2} + \\ & +[|A|^2\varphi^{2\nu-4}(\nu-1)^2 + Re\left[(A\varphi^{\nu-1} - h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]]^2 \times \\ & \times \log\frac{|A|^2\varphi^{2\nu-4}(\nu-1)^2 + Re\left[(A\varphi^{\nu-1} - h\mu^2)^*(\nu-1)(\nu-2)A\varphi^{\nu-3}\right]}{\Lambda^2} + \\ & +N_m N_f h^2(h\varphi^2 + Re(A\varphi^{\nu-1} - h\mu^2))^2 \log\frac{h^2\varphi^2 + Re(A\varphi^{\nu-1} - h\mu^2)h}{\Lambda^2} - \\ & -2N_m N_f h^4\varphi^4 \log\frac{h^2\varphi^2}{\Lambda^2} + N_m N_f h^2(h\varphi^2 - Re(A\varphi^{\nu-1} - h\mu^2))^2 \times \\ & \times \log\frac{h^2\varphi^2 - Re(A\varphi^{\nu-1} - h\mu^2)h}{\Lambda^2}\Big), \end{aligned} \quad (3.23)$$

$$\bar{V}_1^T(\varphi) = -\frac{\pi^2 T^4}{24}\left((N_f+N_m)^2-1\right) + \\ +\frac{T^2}{24}[3(N_f^2-1)|A|^2\varphi^{2\nu-4}+3|A|^2\varphi^{2\nu-4}(\nu-1)^2+8N_mN_fh^2\varphi^2]. \quad (3.24)$$

Similarly, for the quark direction:

$$V_{\rm tree}(Q) = N_e h^2\mu^4 + N_m h^2(Q^2-\mu^2)^2, \quad (3.25)$$

$$V_1^0(Q) = \frac{1}{64\pi^2}\Bigg(-2N_mN_eh^4Q^4\log\frac{h^2Q^2}{\Lambda^2}+N_mN_eh^4(Q^2+\mu^2)^2\log\frac{h^2(Q^2+\mu^2)}{\Lambda^2}+ \\ +N_mN_eh^4(Q^2-\mu^2)^2\log\frac{h^2(Q^2-\mu^2)}{\Lambda^2}+N_m^2h^4(3Q^2-\mu^2)^2\log\frac{h^2(3Q^2-\mu^2)}{\Lambda^2}- \\ -4N_m^2h^4Q^4\log\frac{2h^2Q^2}{\Lambda^2}+(N_m^2-1)(h^2(\mu^2-Q^2)+4g^2Q^2)^2\times \\ \times\log\frac{h^2(\mu^2-Q^2)+4g^2Q^2}{\Lambda^2}+(h^2(\mu^2-Q^2))^2\log\frac{h^2(\mu^2-Q^2)}{\Lambda^2}- \\ -(N_m^2-1)(4g^2Q^2)^2\log\frac{4g^2Q^2}{\Lambda^2}\Bigg), \quad (3.26)$$

$$\bar{V}_1^T(Q) = -\frac{\pi^2T^4}{24}((N_f+N_m)^2-1)+\frac{T^2Q^2}{6}[3N_mN_eh^2+2N_m^2h^2+g^2+5(N_m^2-1)g^2]. \quad (3.27)$$

With these explicit forms for the finite temperature effective potential, we will be able to draw some conclusions about the nature of the phase-transitions in the next section.

Armed with these expressions and the mass-matrices from the previous sub-sections, we can calculate the explicit forms for the free energy.

4. Cooling and the emergence of different phases

We want now to understand what happens to the mesons and squarks φ, Q during the evolution of the universe, as we cool down from a high temperature.

In particular, we want to know the phase structure. If the phase transition in the quark direction happens at a higher temperature than in the meson direction, then we have at least some reason to believe that we will eventually end up in the susy-breaking phase.

Phase transitions are characterized by a critical temperature T_c. By definition, the critical temperature T_c for a second order phase transition is the temperature at which the second derivative of $V(\varphi, Q, T)$ at the origin, in one of the field directions, changes sign from positive to negative. When this happens, the local minimum (at the origin) in that direction becomes a local maximum and a new minimum forms at some finite field value. As a consequence, the vacuum at the origin becomes unstable, a phase transition takes place, and the fields evolve to the newly formed minimum. Of course, again, we emphasize that we are doing an equilibrium analysis, but we believe that this is enough to give a preliminary, heuristic picture of the field history.

We find that T_c^Q in the quark direction is given by

$$(T_c^Q)^2 = \frac{12\mu^2}{3N_e+2N_m+\frac{g^2(1+5(N_m^2-1))}{h^2N_m}}.$$

On the other hand, in the meson direction, we see that there is a minimum away from the origin at this temperature, but, the local minimum at the origin is still there. At some temperature T_c^φ these two minima will become degenerate ($V(0,T_c^\varphi) = V(\varphi_m, T_c^\varphi)$), but there still is a potential barrier between them. Tunneling through the barrier can start when the temperature hits T_c^φ, but this phase transition is first order as opposed to the second-order phase transition in the quark direction. The critical temperature for the first order phase transition in the mesons direction turns out to be

$$(T_c^\varphi)^2 \sim \left(\frac{24N_f}{\pi^2\left((N_f+N_m)^2-1\right)}\right)^{\frac{1}{2}} h\mu^2 + O(h).$$

We notice that in the meson direction the origin is always a local minimum for every temperature, even zero temperature. In fact expanding the 1-loop effective potential at zero temperature $V(\varphi, Q, T=0)$ around the origin we find

$$V(\varphi, Q, T=0) \sim h^2\varphi^2,\ \varphi \sim 0.$$

Finite temperature effects do not change the fact the the origin is still a local minimum in the meson direction. In this case we have

$$V(\varphi, Q, T) \sim T^2h^2\varphi^2,\ \varphi \sim 0.$$

Because the phase transition in the meson directions is first order, it is accomplished through quantum tunneling processes and hence is much more strongly suppressed than the classical phase transition in the quark directions.

To gain a better understanding of the phase transition in the quarks direction we studied the finite temperature effective potential for every value of ϕ and Q close to the critical temperature T_c^Q. The result of this analysis is plotted in figure 5 From the shape of the effective potential around the origin we immediately realize that the flow of the vev happen in the Q direction and it is not possible for the vev to flow in the ϕ direction.

We are now in the position to form an idea about the phase history as the universe cools down. Let's suppose that we are starting at a temperature $T \gg T_c$. We could for example be in the reheating phase after inflation. At this temperature, the origin of field space is a minimum for the finite temperature effective potential $V(\varphi, Q, T)$, figure 1. This is qualitatively plausible, since the massless fields make the biggest contribution to entropy. We also make the assumption that when we start off at this high temperature, the mesons φ and the quarks Q are localized around the origin of field space: $\varphi = 0, Q = 0$ when $T \gg T_c$.

As the temperature decreases to $T = T_c^Q$, the curvature of the effective potential $V(\varphi, Q, T)$ at the origin becomes negative in the Q direction but it stays positive in the φ direction:

$$\left(\frac{\partial^2 V}{\partial Q^2}\right)_{\varphi=0,Q=0,T=T_c} < 0,$$

$$\left(\frac{\partial^2 V}{\partial \varphi^2}\right)_{\varphi=0,Q=0,T=T_c} > 0.$$

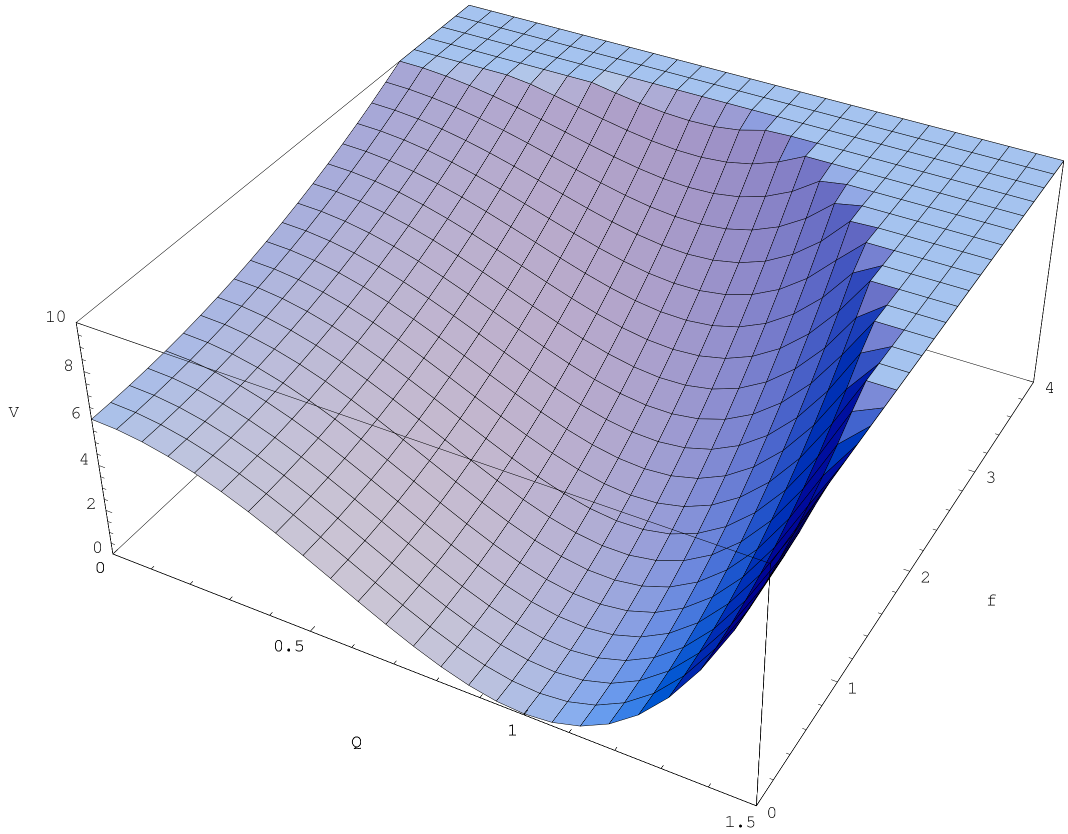

Figure 5: Effective potential for every values of ϕ and Q for $T \sim T_c^Q$

Also, at $T = T_c^Q$, a new minimum Q_m forms in the Q direction, see figure 2. As a consequence, a phase transition occurs and the fields move to the newly formed minimum Q_m. As the temperature of the universe continues to decrease, we eventually arrive at $T \sim 0$, see figure 3, and the minimum Q_m becomes the (meta-stable) non-supersymmetric vacuum $Q_m^0 = \begin{pmatrix} \mu \mathbb{I}_N \\ 0 \end{pmatrix}$. On the meson side, as the temperature drops, the minimum φ_m becomes the supersymmetric vacuum $\varphi_m^0 = \frac{\mu}{h} \frac{1}{\epsilon^{(N_f-3N)/(N_f-N)}}$, but thankfully, phase-transition into the susy phase is suppressed by tunneling at all stages. In writing the expression for the susy-vacuum φ_m^0, we use the Intriligator et al. convention, with $\epsilon \equiv \mu/\Lambda_m$ where Λ_m is the dynamically generated scale of our (infrared) theory. As mentioned earlier, it is the scale of the Landau pole.

Thus, the phase structure of the theory seems to imply that for reasonably tame initial conditions for the scalar quarks and mesons (namely, they start off near the origin of field space), the phase transitions lead us into the susy-breaking vacuum at $T \sim 0$:

$$\varphi = 0, \; Q_m^0 = \begin{pmatrix} \mu \mathbb{I}_{N_f} \\ 0 \end{pmatrix}.$$

JHEP03(2007)107

5. Conclusions and future directions

We found that the phase-structure of the free energy seems to imply that as the universe cools, we end up in the susy-breaking vacuum.[3] In this final section, we point out some caveats and limitations inherent in our study. One of these, is the assumption of thermal equilibrium. By working with the free-energy, we are ignoring the possibility that the fields could evolve and interact with the heat bath. The dynamics of the fields could result in overshoot or undershoot around the vacua. But it seems unlikely that these effects will destroy our conclusion if the scalars are starting at the origin of field space, at least if the evolution of the universe is close to adiabatic. That brings us to a related issue, which is that we don't have a good idea about what are good initial conditions for the scalars, as was discussed in the introduction.

When calculating the effective potential, we focussed on just the meson and squark directions. There is the possibility that we would discover new valleys and slopes if we were to make a map of the potential for all field values. One possibility is to do a perturbative analysis around the quark/meson directions to see whether these directions are indeed troughs and not saddles at all temperatures.

Another issue which could affect our conclusions, even though it is likely to be less important, is the question of the time scales of first order versus second order transitions. It seems likely that the original argument of Intriligator et al. suggesting that the meta-stable vacuum can be made parametrically stable against tunneling, should work (perhaps with a suitable modification) for our case too.

To deal with non-equilibrium situations and initial conditions, one could work in the context of the real-time formulation of finite temperature field theory where we can consider both dynamics and thermal effects. There, the interaction with the heat bath would be encoded in a friction-like term. Preliminary investigations in this direction seems to imply that at least for some choice of scalar initial conditions, we end up in the meta-stable vacuum. We leave a more thorough analysis of this issue for future work.

Acknowledgments

We thank Patrick J. Fox and Jay G. Wacker for useful conversations near the beginning of this work. We would like to thank Uday Varadarajan for giving a talk on the ISS paper and thereby triggering this work, and also for discussions. The research of W.F., L.M. and V.K. was supported by NSF under grant PHY 0071512 and 0455649. The work of C. K. is supported in part by IISN - Belgium (convention 4.4505.86), by the Belgian National Lottery, by the European Commission FP6 RTN programme MRTN-CT-2004-005104 in which C. K. is associated with V. U. Brussel, and by the Belgian Federal Science Policy Office through the Interuniversity Attraction Pole P5/27. The work of M. T. is supported in part by CAPES.

[3] When we were about to submit our paper, we became aware of an article that studies the same subject with similar conclusions [20].

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JHEP03(2007)107